COMPOSITIONAL MEASURES

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Boolean background

▹ *Boolean algebra:*
The interpretation domain **B** is a complete Boolean algebra **B** = <B, $\sqsubseteq$, $\neg$, $\sqcap$, $\sqcup$, 0,1>.

▹ $X^+ = X - \{0\}$

▹ *Disjointness:* X *overlaps* iff for some $d_1, d_2 \in X$: $d_1 \sqcap d_2 \in X^+$; otherwise X is *disjoint*.

▹ *Semantic plurality as closure under sum*: $*X = \{d \in B$: for some $Z \subseteq X$: $d = \sqcup Z\}$

▹ *Definiteness operator as presuppositional sum*: $\sigma(X) = \sqcup X$ if $\sqcup X \in X$, $\perp$ otherwise.

▹ *Part set*: $(d] = \{b \in B: b \sqsubseteq d\}$

▹ *Cardinality*

$$\textbf{card}_X(d) = \begin{cases} |(d] \cap X| & \text{if } d \in {*X} \text{ and } X \text{ is disjoint} \\ \perp & \text{otherwise} \end{cases}$$

▹ The set of *X-atoms*: ATOM_X is the set of minimal elements in X^+.
The set of *X-atomic parts* of d: $\text{ATOM}_{X,d} = (d] \cap \text{ATOM}_X$
X is *atomic* iff for every $d \in X^+$: $\text{ATOM}_{X,d} \neq \emptyset$
X is *atomistic* iff for every $d \in X^+$: $d = \sqcup(\text{ATOM}_{X,d})$

▹ *Additive measure functions*
$\mathbb{R}^+$ is the set of real numbers from 0 up, W is the set of possible worlds.
A *additive measure function* is a function μ: $B \times W \to \mathbb{R}^+$ such that
for all $w \in W$: $\mu_w(0) = 0$ and for every countable disjoint subset X of B:

$$\mu_w(\sqcup X) = \Sigma\{\mu_w(x): x \in X\}$$

(We will also call μ_w an additive measure function)

1. COUNTING

View implicit in a lot of thinking about the mass-count distinction,
explicitly defended in Rothstein 2017:

▹ **MC_1:** There is a mass domain and there is a count domain, and they are disjoint.
MC_2: Mass nouns are interpreted in the mass domain, count nouns in the count domain.
COUNT: Counting takes place in the count domain, but not in the mass domain.
MEAS: Measuring takes place in the mass domain, but not in the count domain.

MC_1, MC_2, COUNT: e.g. Link 1983, Landman 1991, Rothstein 2017
MEAS: Rothstein 2017
This talk is about **MEAS**

Problem with **COUNT**: A lot more counting takes place in the mass domain (or with mass noun denotations) than we (or some of us) used to think 40 years ago.

Unproblematic: grammatical shift between mass to count.
Shifted nouns pattern with the grammatical category they shift into:
Singular count noun *pig* in (1) is shifted to a mass noun and mass noun *beer* is shifted to a count noun:

(1) In the Bierhalle, Heinz ate *much pig* and drank *many beers*.

Problematic: *Neat mass nouns* like *livestock*, *pottery, mail* are mass nouns, but pattern with count nouns rather than mass nouns on several tests. E.g. ***count comparison*** (e.g. Bale and Barner 2002):

Assume that on the farm there are 20 cows outside and 1000 chickens inside.
The cows yield 5000 kg of meat, the chickens yield 400 kg of meat.

(2) a. Most *farm animals* are inside in summer. TRUE
b. Most *livestock* is inside in summer. TRUE
c. Most *meat* comes from animals that are inside in summer. FALSE

Plural count noun phrase *farm animals* in (2a) has only a count comparison reading (cardinality).
Mess mass noun *meat* in (2c) has only a measure comparison reading (weight).

Neat mass noun *livestock* in (2b) prominently has a count comparison reading.
But neat mass nouns allow both count comparison and measure comparison. Rothstein 2017:

(3) a. Why did Mary come home later from the post office than Jane?
Mary had *more mail* to bring home. She had to fill out a separate form for each letter. COUNT
b. Why did Jane take a taxi to come home, while Mary didn't?
Jane had *more mail* to bring home. She had three bulky and heavy parcels. MEASURE

Landman 2020: languages like Dutch and German allow count comparison readings to be triggered contextually even for mess mass nouns like *meat*. Examples in Landman 2020.

Rothstein 2017 is well aware of these cases. But she adopts the COUNT constraint:

> Counting is putting individual entities into one-one correspondence with the natural numbers, while measuring is assigning a measure value to a quantity on a dimensional scale independent of the internal structure of that quantity. [p. 106]
> In the count domain, quantity comparisons are always in terms of cardinality, since the semantically encoded atomic structure of the predicate makes this the only parameter for evaluation available. [p. 141-142]
> [49] a. Counting is a an operation on count noun denotations.
> b. Measuring is an operation on mass noun denotations. [p. 142-143]

So how can we do count-comparison in the mass domain in Rothstein's theory?
Rothstein's answer [reformulated in terms of measure functions]: She assumes:
▻ **ADD**: The measure functions that are relevant in the mass domain are additive measure functions. [Krifka 1989]

Let AT_w be a set of atoms in a contextually given Boolean structure.
Since AT_w is disjoint relative to the relevant Boolean order, it is easy to show that:
$\lambda w.\ \mathbf{card}_{AT_w}$ is an additive measure function.

Rothstein's argument: $\lambda w.\ \mathbf{card}_{AT_w}$ is an additive measure function, on a contextually salient atomic concept AT.
Hence it is available in the mass domain.
Hence it need not be a surprise if languages allow it to be made available for mass nouns in contexts where AT has been made sufficiently salient, like neat mass nouns.
Rothstein's claim: This use of $\lambda w.\ \mathbf{card}_{AT_w}$ in the mass domain is *not* counting:

> We don't [*need*] to use these scales [= $\lambda w.\ \mathbf{card}_{AT_w}$] to compare cardinalities, and in fact counting is a way of not doing so. If we compare how many cats John and Mary each have by counting, and we count to ten in the first case and seven in the second case, then we know that Mary has more cats, because we know that ten is more than seven. [p. 137]

Criticism:
If we compare how many cats John and Mary each have, we have to count how many cats John has and we have to count how many cats Mary has. In order to count how many cats John has we have to determine two things:

▻ **ONE:** Determine what counts as *one* for CATS: this is set CAT_w
▻ **DISJOINT:** Make sure that the relevant context is one where *the set* CAT_w *of things that count as one* is *disjoint* ***in the way that is relevant for counting***.

Given these two conditions, objects in $*CAT_w$, the closure under sum of CAT_w, are 'put in one-one correspondence with the natural numbers' by the cardinality function $\lambda w.\ \mathbf{card}_{CAT_w}$.

My conclusion:
The idea that matching pluralities with the natural numbers is something different from applying the function $\lambda w.\ \mathrm{CARD}_{CAT_w}$ is misleading.
Better linguistics assumption:
Count nouns are nouns for which function $\lambda w.\ \mathbf{card}_{N_w}$ is semantically available.
This function can be made available relative to a contextually salient set for mass nouns in different ways.
Hence: I reject assumption **COUNT**.

2. MEASURING

▻ **MEAS**: Measuring takes place in the mass domain, but not in the count domain.

Rothstein's first argument: plural count complements of nominal measures
Observation 1: Nominal measures are intersective

(4) *Ten kilos of flour* $= \lambda x.\mathrm{FLOUR}_w(x) \wedge \mathbf{kilo}_w(x)=10$ = Flour to the amount of 10 kilos is flour

Observation 2: Measure phrases pattern with mass nouns:

(5) I haven't read ✓*much/#many* of the *twenty kilos of books* that we sent.

Rothstein: Observations 1 and 2 show that *books* in *twenty kilos of books* must shift to mass, because $\lambda x.{*}\mathrm{BOOK}_w(x) \wedge \mathbf{kilo}_w(x)=20$ books to the amount of 20 kilo is books, hence count.
Conclusion: Measures operate in the mass domain, not in the count domain.

Criticism: Landman 2020 accepts observation 2, but argues that observation 1 is irrelevant.
Landman 2020: it is not the interpretation of the expression *flour/potatoes* in (4/5) that determines the mass/count nature of the measure phrase,
but the interpretation of the ***head*** of the measure phrase, i.e. *kilo*, and this head is mass.

Iceberg semantics background

▻ An *i-set* is a set $X = <\mathbf{body}(X), \mathbf{base}(X)>$ with $\mathbf{body}(X), \mathbf{base}(X) \subseteq B$
and $\sqcup(\mathbf{body}(X)) = \sqcup(\mathbf{base}(X))$
and $\mathbf{body}(X) \subseteq {}^*\mathbf{base}(X)$

X is *count* iff **base**(*X*) is disjoint, otherwise *X* is *mass*.
X is *neat* iff **base**(*X*) is atomistic and $\text{ATOM}_{\mathbf{base}(X)}$ is disjoint, otherwise *X* is *mess*.
[see Landman 2020 for the real definitions taking null i-sets into account]

▻ *Intensions* are functions from worlds to i-sets
Intension f is *count* iff for every $w \in W$: f_w is count
Intension f is *neat* iff for every $w \in W$: f_w is neat
Intension f is *mess* iff for every $w \in W$: f_w is mess
Intension f is *mass* iff f is not count

Compositional theory of mass/count/neat/mess in terms of bases of noun phrase interpretation:

▻ *Head Principle*: the **base** of the interpretation of a complex NP is
the set of all parts of the **body** of the interpretation of that NP
intersected with the **base** of the interpretation of the head of that NP.

Iceberg Semantics: interpretations of noun phrases are icebergs, pairs of sets consisting of
-a **body**, which is the interpretation familiar from Boolean semantics for plurals and mass nouns.
-a **base**, which is a set that generates the **body** under sum.
-The mass-count nature of the noun phrase is determined by the **base**:
if the **base** is a (contextually) disjoint set, the interpretation is count, otherwise mass.

Plural count noun *potatoes* → $<^*\text{POTATO}_w, \text{POTATO}_w>$
The **base** is disjoint set POTATO_w, the set of potatoes that count as one. This i-set is count.

Let in context w all our poultry consist of turkeys:
Neat mass noun *poultry* → $<^*\text{TURKEY}_w, {}^*\text{TURKEY}_w>$
The **base** is the closure under sum of disjoint set TURKEY_w This set is not disjoint.
The i-set is mass.

Landman 2020: Semantics of nominal measure *kilo*: (ignoring fine details)
Body: measure function $\mathbf{kilo}_w$
Base: $\lambda x.\mathbf{kilo}_w(x) \leq \mathbf{min}_{\text{kilo},w}$
The set of objects in the domain whose weight in kilos is less than a small contextual value.

The compositional semantics derives for *ten kilos of potatoes*:
Body: potatoes to the amount of 10 kilos
Base: The set of arbitrary parts of the sum of potatoes weighing less than the contextual value.

Fact: this base is not disjoint. hence this i-set is mass, despite the intersective **body**-semantics.

Rothstein's second argument: count/measure comparison for plural nouns

The second argument is the observation that plural count nouns only allow count comparison, not measure comparison.

(6) Most cats are white.

Suppose we have two black cats, Shunra and Pim, that are enormous, their combined weight is much more than the combined weight of our three little white cats, Emma, Bruno and Sasha.

(6) obviously has a count comparison reading on which it is true,
and *equally obviously* (6) *lacks* a measure comparison reading on which it is false:
the weight of the cats is irrelevant for the truth conditions of (6)

Rothstein: *cats* is a plural count noun, whose interpretation is in the count domain,
COUNT says that **card** is defined there, MEAS says that measures are undefined there.

Puzzle: Nothing in the semantic structures of the mass domain and the count domain makes you expect this: they have similar Boolean structures and there is, of course, no problem with defining additive measures on atomic Boolean algebras.
In fact, given the parallels between mass nouns and plural count nouns that have been stressed in the semantic literature since Link 1983, the lack of measure readings is rather baffling.

MEAS is a not particularly insightful stipulation. Can we do better?

TWO RED HERRINGS
Grammatical account 1: Rothstein 2017
The lexical semantics of count nouns makes **card** lexically available (even as a null-classifier). Since this one is lexically available, the grammar determines that this is the one that must be picked in comparison.

Criticism:
Reasonable: lexical availability makes **card** a measure comparison ***can*** pick.
Not clearly reasonable: this is the measure that measure comparison ***must*** pick.
Many things that are lexically available can be skipped over when the semantics is compatible.

Grammatical account 2: Discussed in Rothstein 2017
For several grammatical concepts only one can be grammatically specified at a time on a single head. (e.g. thematic roles)
If **card** is a measure that is lexically specified on count nouns, and there is a requirement of only one measure, then that explains the lack of measure comparison readings for plural count nouns.

However, this general principle does not seem to exist.

(7) The magic will only work at the right concentration, when volume and weight are in balance. That is, that is, the bottle must contain ***50 ml*** and ***20 grams*** of polyjuice potion.

Here two different measures associate felicitously with the same head.

3. MEASURE COMPARISON

(8) a. Most cats purr.
b. Most mud is brown.

In (8a), the interpretation of *most* in world w combines with the iceberg intension of *cats* and the intension of *purr*:

CATS $= \lambda w.<{*}CATw, CAT_w>)$
PURR $= \lambda w.PURR_w$

On the (standard) reading of *most* I will use as an example here it compares in w:

$\sigma({*}CAT_w) \sqcap \sqcup({*}CAT_w \cap PURR_w))$	The sum of the cats that purr
and	
$\sigma({*}CAT_w) - \sqcup({*}CAT_w \cap PURR_w))$	The relative complement of that, i.e. the sum of the cats that don't purr

Hence, for iceberg intension f and property V: it compares:

Notation:

$[f{+}V]_w$	$\sigma(\mathbf{body}(f_w) \sqcap \sqcup(\mathbf{body}(f_w) \cap V_w))$	The sum of the parts of **body**(f_w) that have V
	and	
$[f{-}V]_w$	$\sigma(\mathbf{body}(f_w) - \sqcup(\mathbf{body}(f_w) \cap V_w))$	The relative complement of that sum.

I will now make a central ***grammatical*** assumption:

▻ **Base-linked measures:**
The comparison in the semantics of *most* involves a ***base-linked measure***,
a measure that is linked to the base of the interpretation of the complement noun:
i.e. the comparison with *most* involves a function $\boldsymbol{\mu}$, with:
$\lambda w \lambda f.\ \boldsymbol{\mu}_{\mathbf{base}(f_w)}$

The theory introduced below will put restrictions on what functions can occur as base-linked measure functions.
This will (maybe not surprisingly) be exactly the class of measure functions that can be the interpretations of nominal measures, measures in nominal measure phrases (like *kilo* in *three kilos of apples*). So I propose:

▻ **Nominal measures:**
Nominal measures are measures that can be **base**-linked.

[Note: The fact that the measure can be **base**-linked in comparison does not mean that this feature is used in all its uses. In fact, as we have seen, it is not used in measure phrases: *kilo* in *three kilos of apples* is a nominal expression that has ***its own*** measure base.]

With this, the interpretation schema for the relevant interpretation of *most* is:

▻ *most* → $\lambda w \lambda f \lambda V.\ \ \boldsymbol{\mu}_{\mathbf{base}(f_w)}([f{+}V]_w) > \boldsymbol{\mu}_{\mathbf{base}(f_w)}([f{-}V]_w)$
The $\boldsymbol{\mu}_{\mathbf{base}(f_w)}$ measure value of $[f{+}V]_w$ is bigger than that of $[f{-}V]_w$
The measure value of F plus V is bigger than that of f minus V.

This unified schema for the semantics of *most* allows us to locate our problem:
We can now think about what $\lambda w.\boldsymbol{\mu}_{\mathbf{base}(f_w)}$ can be:

Interpretation possibilities for $\lambda w.\boldsymbol{\mu}_{\mathbf{base}(f_w)}$**:**

- $\lambda w.\mathbf{card}_{\mathbf{base}(f_w)}$ Restriction: **base**(f_w) is disjoint (i.e. f is count)
- Other count options, like :
  $\lambda w.\mathbf{card}_{\mathbf{ATOM}_{\mathbf{base}(f_w)}}$ Restriction: $\mathbf{ATOM}_{\mathbf{base}(f_w)}$) is disjoint (i.e. f is neat)
  etc.
- μ, where μ is a measure sortally appropriate for f
  Restriction relative to **base**(f_w): **to be found out below**

We can reformulate our problem in terms of $\lambda w.\boldsymbol{\mu}_{\mathbf{base}(f_w)}$**:**

▻ What is it about the bases of count nouns that makes **card** the only available choice for μ in $\lambda w.\boldsymbol{\mu}_{\mathbf{base}(f_w)}$?

Since, **card** *is* an available measure, this question becomes:

▻ ***The fundamental question*****:**
What is it about the bases of count nouns that make measures unavailable as choice for μ in $\lambda w.\boldsymbol{\mu}_{\mathbf{base}(f_w)}$**?**

4. COMPOSITIONAL MEASURES

The analysis is based on two ideas: **base**-compositionality and **base**-corroboration.

4.1. BASE-ADDITIVITY

Base-compositionality is Iceberg semantics: Iceberg semantics gives a compositional theory of the notions *mass, count, neat, mess* in terms of the **bases** of interpretations:

Base: compositionality: An NP-denotation is *count* if its **base** is disjoint, otherwise *mass*. An NP-denotation is *mess* if its **body** is generated (under sum) by the set of minimal **base**-elements and that set is disjoint, otherwise *mess*. Head Principle: The **base** of the denotation of a complex NP is the part-set of the **body** of its denotation intersected with the **base** of the denotation of its head.

I proposed above that the measure function is **base**-linked: $\lambda w.\boldsymbol{\mu}_{\mathbf{base}(f_w)}$
I propose now that this means that its relevant semantic properties of base-linked measures are **base**-compositional in the same way as the notions of mass/count/neat/mess.

▻ **Base-determined measuring:**
For every d ∈ ***base**(f_w): the measure value $\boldsymbol{\mu}_{\mathbf{base}(f_w)}(d)$ is determined by the measure values of parts of d in **base**(f_w).

The measure value of object d generated under sum by the **base** is determined by the measure values of parts of d **in the base**.

One can cook up intricate ways in which an arbitrary measure could satisfy this, but from a linguistic point of view where we are concerned with ordinary concepts there is really only one natural way in which this principle can be understood:

▹ **Base-additivity of base-linked measuring**:
f_w is ***base**-additive* for **μ** iff
For every d ∈ ***base**(f_w): there is a *countable disjoint* set X ⊆ **(**d**]** ∩ **base**(f_w)
such that d = ⊔X and $\boldsymbol{\mu}_{\mathbf{base}(f_w)}(d) = \Sigma(\{\boldsymbol{\mu}_{\mathbf{base}(f_w)}(x): x \in X\})$

The measure value of each object d generated under sum by the **base** can be calculated as the sum of the measure values of the elements of some countable disjoint set of **base**-parts of d.

We assumed above that nominal measures are **base**-linked.
We now assume that **base-**linking is defined (exactly how to be determined below) in terms of **base**-additivity.
We derive from this an obvious conclusion:

▹ **Additivity:**
Base-linking of nominal measures implies that nominal measures are additive.

This is Krifka 1989's observation.

(9) a. *Nominal measure*:

Ten kilos of apples	λx.*APPLE$_w$(x) ∧ **kilo**$_w$(x) = 10	
	Objects that are sums of apples and weigh ten kilos	
	= ✓Apples *to the amount of* 10 kilos	ADDITIVE
b. *measure adjunct*:	λx.WATER$_w$(x) ∧ ℃$_w$(x) = 60	
60 degree water	Objects that are water and whose temperature is 60 ℃	
	= Water of sixty degrees	NOT ADDITIVE
	≠ Water *to the amount of* 60 degrees	

Given the semantic similarities between mass nouns and plural count nouns,
there isn't actually a real semantic difference between the *denotation* of *ten kilos of apples* and the denotation of *60 degree water*, *water of 60 degrees*.

But this denotation is not a possible interpretation for the nominal measure
#60 degrees of water: nominal measures are only *felicitous if* you can reformulate that denotation as an *additive amount statement*.

4.2. BASE-CORROBORATION

The heart of the analysis concerns corroborations of measuring. My assumptions go in two steps:
1. Measuring requires corroboration.
2. **Base**-linked measures require corroboration to be **base**-linked.

How do you check that you got a measure value right? Well, of course, by calculating it again. But also, by corroborating it: by calculating the value in a different way.
This, in fact, we do both for measuring and for counting.
-We have a liquid divided over different vessels, we calculate the volume of each and add up. To check, we may get a fixed volume vessel and check how many times this volume can be filled.
-Or for counting: we count all our pennies and get a number. Then we divide them into piles of ten and count those. This is what I call corroboration here:

Corroboration of additive measures:

1. **Additive counting:** Calculate the measure value of d by partitioning d into a countable disjoint set of parts of which you know the measure values and add up those values.
2. **Corroboration:** Calculate the measure value of d again by partitioning d into ***a different*** countable disjoint set of parts, add up those values, and check you get the same value.

I take corroboration to be an essential feature of measuring and calculating.
This proposal gets linguistic bite by assuming for **base-**linked measures **base**-compositionality ***also for corroboration***:

▻ **Base-corroboration of base-linked measuring**:
f_w is ***base**-corroborative* for **μ** iff
For every $d \in {}^*\mathbf{base}(f_w) - \mathrm{ATOM}_{\mathbf{base}(f_w)}$:
there are two ***distinct*** *countable disjoint* sets $X_1, X_2 \subseteq (d] \cap \mathbf{base}(f_w)$
such that $d = \sqcup X_1 = \sqcup X_2$
and $\boldsymbol{\mu}_{\mathbf{base}(f_w)}(d) = \Sigma(\{\boldsymbol{\mu}_{\mathbf{base}(f_w)}(x_1): x_1 \in X_1\}) = \Sigma(\{\boldsymbol{\mu}_{\mathbf{base}(f_w)}(x_2): x_2 \in X_2\})$

The measure value of each object d generated under sum by the **base** can be calculated as the sum of the measure values of the elements for ***two (sufficiently) distinct*** countable disjoint sets of **base**-parts of d.

(The exception are the **base-**atoms which, of course, in ***base(**f_w) are only the sum of themselves.)

▻ **Consequence:** If f_w is ***count*** then f_w is not **base**-corroborative for any **base**-linked measure **μ.**

Reason: if f_w is count then **base**(f_w) is disjoint and generates ***base**(f_w) under ⊔.
But then every element in ***base**(f_w) is the sum of *exactly one* set of **base** elements and **base**-corroboration is not possible.

So this is promising: **base**-corroboration is not possible for the interpretations of count nouns.
So it always possible for the interpretations of mass nouns?

4.3. CLOSE BUT NO SIGAR

The question is hard to answer in general.
In Landman 2020 I give suggestions for Iceberg semantic analyses of mass nouns along a scale of interpretations. I will indicate how well these suggested interpretations do.
(Answer: see the title of this section.)

1. Homogeneous mass nouns like *time* (mass denotations closed under parts)
-Model: complete atomless Boolean algebra of periods.
-Moments: Fix in context a partition of moments of time: intervals of some size r that in the context count as not further divided (see Landman 2020)
-**base**($time_w$) is the set of all intervals of at most as small as moments.
This set is not disjoint, hence $time_w$ is mass.

▻ Fact: if d ∈ ***base**($time_w$) and **μ** additive and some countable disjoint subset of **base**($time_w$) adds up to μ(d) then this partition can always be refined to a different **base**-partition.

Problem: It is not guaranteed that every element *is* the sum of a *countable* subset of elements of **base**($time_w$) in the first place.
And there is no good reason to impose that on all mass noun denotations.
I deal with this problem in the proposal below.
So we will concentrate in the next cases on corroboration only. Base-corroboration, we see, is satisfied here.

2. Mass nouns with contextual 'smallest parts' like *meat*
-In context we set what is roughly the smallest size of pieces of meat that themselves count as meat. Say, what you can get with the finest cutting knife machine (allowing some variation in size). One cutting is a partition, but any similar partition (like moving the cutter a bit to the side over the meat) cuts into pieces that count as meat.
base($meat_w$) is the union of all those partitions, which is not disjoint, hence $meat_w$ mass.

▻ Fact: if d ∈ ***base**($meat_w$) and **μ** additive there are, by the construction **many** partitions in **base**($meat_w$) that add up to **μ**(d). So base-corroboration is satisfied.

3. Mass nouns with 'atoms' like *water*
-While in context we can think of *water* as being generated like *meat* from drops of water, I argue in Landman 2020 that even the 'scientific' view of water as H_2O allows a mass perspective.
The idea for that is: our water puddle is not partitioned just into water molecules, but into water molecules and *space* (pairs of sums of water molecules and regions of space).
We partition the space into regions containing exactly one water molecule, and we let **base**($water_w$) be the union of those partitions. This set is not disjoint, hence $water_w$ is mass.

I discuss two models in Landman 2020:
Atomless: The **base**-elements consist of a water molecule and a region bigger than the eigenspace of that water molecule.
Atomic: The **base**-elements consist of a water molecule and a region bigger or equal to the eigenspace. (Here the molecules-eigenspace pairs are **base** atoms.)

▻ Fact 1: **Base**-corroboration holds for the atomless model:
If d ∈ ***base**($water_w$) and **μ** additive and some countable disjoint subset of **base**($water_w$) adds up to μ(d) there are **many** partitions in **base**($water_w$) that add up to **μ**(d).
Base-corroboration is satisfied.
▻ Fact 2: **Base**-corroboration fails for the atomic model:
If **base**($water_w$) is atomic and d_1, d_2 are water molecule-eigenspace pairs ,
then $d_1 \sqcup d_2$ ∈ ***base**($water_w$) **–** **base**($water_w$) and is uniquely the sum of d_1 and d_2.
The atomic model doesn't quite satisfy base-corroboration.

4. Sum neutral neat mass nouns like *poultry*
I proposed that for these neat mass nouns the **base** is identical to its closure under sum.
Say, if all the poultry is turkeys, then **base**(*poultry*$_w$) = *TURKEY$_w$. This set is not disjoint, hence *poultry*$_w$ is mass, but generated by a disjoint set (TURKEY$_w$), hence *poultry*$_w$ is neat.
Here every element in **base**(*poultry*$_w$) **–** TURKEY$_w$ is the sum of **base**-elements in two different ways, namely as a sum of **base**-atoms (since that set generates the **base**), and as a sum of itself, hence:

▻Fact: If d ∈ **base**(*poultry*$_w$) and **μ** additive and besides d itself some countable disjoint subset of **base**(*poultry*$_w$) adds up to μ(d) there are at least two distinct partitions in **base**(*poultry*$_w$) that add up to **μ**(d).

For neat mass nouns with a finite set of **base**-atoms like *poultry,* this is of course satisfied.
So base-corroboration is satisfied.

5. Group neutral neat mass nouns like *pottery*

These are aggregate nouns where a sum of base-atoms may also count as one.
So in out shop **base**(*pottery*$_w$) could be the items that are sold independently
{the bonbon-tray, the cup, the saucer, the teapot, *the cup and saucer*, *the teaset*}
Here **base**(*pottery*$_w$) is not disjoint, hence *pottery*$_w$ is mass.
But **base**(*pottery*$_w$) is generated by the set of **base** atoms, hence *pottery*$_w$ is neat.

But here we see the same problem as what we saw for the atomic model for *water*:
the bonbon-tray ⊔ the cup ∈ ***base**(*pottery*$_w$), but it is not in **base**(*pottery*$_w$)
and it is only in one way the sum of **base**-elements.
So **base**-corroboration is not satisfied here.

6. A general problem with neat mass nouns

Intension f is *count* iff for every w: f_w is count, otherwise mass.
Intension f is *neat mass* iff for every w: f_w is neat and not for every w: f_w is count.

This means that neat mass intensions ***allow*** instances f_w where the denotation is count.
And it doesn't seem reasonable to forbid that possibility in context for neat mass nouns.
That means that for this reason too **base-**corroboration is not guaranteed for neat mass nouns.

4.4. BASE-MEASURE FLEXIBILITY

The relevant difference between mass noun phrases and count noun phrases is not a question of extension (f_w) but of intension ($\lambda w.f_w$). The difference at the intension level is **flexibility:**

A▻ **Base-inflexibility**
The bases of count noun intensions cannot be stretched.

-Count noun phrases with a *conceptually* disjoint **base**, like *penny*.
Base of count noun *pennies*: single pennies, not groups of pennies.
-You stay within the meaning of *penny* if you add more pennies to the **base**.
-You ***do not*** stay within the meaning of *penny* if you add groups of pennies to the **base**.

-Count noun phrases with a *contextually* disjoint **base**, like *portions of soup*
six portions of soup partitions the soup in context into disjoint portions.
-We can *change the context* and count the same soup as *twelve portions of soup*, then we partition into twelve disjoint portions.
-We do not stay within the meaning of *portion of soup* if we take **as base** the union of the six portions of soup and the twelve portions of soup.

i.e. You can *only stretch* in context the **base** by adding *more soup* and portioning it, not by counting more of what there is as *portion of soup* (the same for nouns like *fence*).

B ▻ **Base-flexibility**
The bases of mass noun intensions can be stretched.

As we have seen, a mass noun denotation may not satisfy **base**-additivity or **base**-corroboration.
But the **bases** for mass noun intensions are ***contextually flexible***:
You can always let the context stretch such a base to a **base** that **does** satisfy **base**-additivity and **base**-corroboration **and stay within the intension of the mass noun**.

▻ **Base-stretching**
v stretches the **base**(f_w), v $\sim_f$ w iff v at most differs from w in that **base**(f_w) ⊆ **base**(f_v)

In particular, this means that: ***base**(f_w) = ***base**(f_v) and **body**(f_w) = **body**(f_v)

So if w $\sim_f$ v the only difference between w and v is that **base**(f_v) may extend more into ***base**(f_w) (= ***base**(f_v)) than **base**(f_w) does.

▻ **Base-measure flexibility**
Let f be an intension and **μ** a base-linked measure.

f is **μ**-***base*** *measure flexible* iff for every w ∈ **dom(**f): there is a v ∈ **dom**(f): w $\sim_f$ v and f_v is ***base****-corroborative* for **μ**

For world w for which f is defined, f_w may be not **base**-corroborative, but there always is a world v where f is defined and f_v *is* **base**-corroborative, and v at most differs from w in that **base**(f_v) stretches **base**(f_w) inside ***base**(f_w).

1. Base additivity
time: **base**-corroboration was not the problem, but **base-**additivity was.
There may be elements in ***base**($time_w$) that are only the sum of continuous many moments, not countably many.

But obviously we can divide time into a less refined countable partition of moments without leaping out of the meaning of mass noun *time*.
This makes mass intension λw.$time_w$ **base**-measure flexible with respect to additive measure **duration**.

This holds generally for mass nouns and what is at stake is *cumulativity* and *flexibility of* ***base***.
Count nouns like *penny* are not cumulative: a pile of *pennies* is not itself a *penny*.

Plural nouns and mass nouns are cumulative: a groups of turkeys is poultry, and an aggregate of pottery is pottery, bigger stretches of time are time.

But plural nouns like *pennies* have the same **base** as their corresponding singular noun *penny*.
What mass nouns have is what plural nouns lack: **base**-flexibility:
the bases of mass nouns can be stretched if the context so wants it.
This means that stretching the **base** up to allow for a countable partition of the top element in terms of **base** elements is contextually possible.

2. Base-corroboration

Exactly the same argument applies to the other case, like that where a sum of two pottery items was only in one way the sum of two **base** elements.
We can liberalize our policy about what we are willing to sell as one item: if you want me to sell you a bonbon-tray-plus-cup, sure I'll make that an item that I sell and for a good price, I have no pride (my God, I sell it even the pattern clashes violently, what the hell...)

Conclusion:
1. All the mass nouns and noun phrases that I studied in Landman 2020 have intensions that are arguably **base** measure flexible with respect to appropriate measures.
2. Count nouns never have intensions that are **base** measure flexible with respect to any appropriate measure.

5. THE PROPOSAL

▻ **The proposal:**
1. Only measures that can be **base**-linked can be nominal measures.
2. Comparison in the semantics of *most* requires a **base**-linked measure.
3. **base**-linked **card** is defined in terms of **base**-disjointness, but can generalize to linking to contextually salient disjoint sets when the **base** is not disjoint.
4. **base** linked measures, besides **card,** are measures that noun intensions can be **base**-measure flexible on.

Interpretation possibilities for $\lambda w.\boldsymbol{\mu}_{\mathbf{base}(f_w)}$:
- count options: as above
- μ, where μ is a measure sortally appropriate for f
Restriction: f is μ-base measure flexible

▻ **Conclusions:**
Landman 2020:
2020
1. Comparison in *most*[N,V] can be in terms of **card** for *plural count nouns*.
2. Comparison in *most*[N,V] is not possible for *singular count nouns* independently because of the semantics of *most*: hence their interpretation downshifts to mass (as in: *most **hippopotamus** is eaten in Afrika*).
3. Comparison in *most*[N,V] can be in terms of **card** relative to salient disjoint sets for neat mass nouns N or even for some mess mass nouns in Dutch and German.

In this talk we derive
2022
4. Nominal measures are additive.
5. Comparison in *most*[N,V] can not be in terms of *measures* for plural count nouns, because plural count noun intensions are not **base-measure flexible** with respect to

any sortally appropriate measure**.**

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